

## CLAIMS

We claim:

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1. A method for optimizing pricing and capacity for bandwidth management using a computer, comprising the steps of:

inputting a mean and a variance of real usage for each of a plurality of customer classes;

inputting price and demand curve data which determines an arrival rate for each customer class;

inputting a number of existing customers in each customer class;

inputting a bandwidth wholesale cost;

generate a computer model for an optimization problem subject to a plurality of predetermined chance constraints;

solving said optimization problem using said computer to determine an amount of bandwidth to be purchased in a time period at a given price for an expected number of new customers in order to maximize profit; and

outputting said amount of bandwidth to be purchased and said expected number of new customers.

2. A method for optimizing pricing and capacity for

bandwidth management using a computer as recited in claim 1 wherein said plurality of predetermined chance constraints comprises:

$$b_{\tau} = b_{\tau-1} + a_{\tau} \quad (\tau = 1, \dots, T) \quad (1)$$

$$L_{i\tau} \leq q_{i\tau} \leq U_{i\tau} \quad (i = 1, \dots, I; \tau = 1, \dots, T) \quad (2)$$

$$\begin{aligned} & \sum_{i|\tau < d_i} [\lambda_{i\tau} \Delta \mu_i^2 + (n_{i\tau} + \lambda_{i\tau} \Delta)^2 \sigma_i^2 + (n_{i\tau} + \lambda_{i\tau} \Delta)^2 \mu_i^2] \\ & + \sum_{i|\tau \geq d_i} [\lambda_{i\tau} D_i (\mu_i^2 + \sigma_i^2)] + (\lambda_{i\tau} D_i \mu_i)^2] - \delta_{\tau} b_{\tau}^2 \leq 0 \quad \forall \tau \end{aligned} \quad (3)$$

and said optimization problem comprises:

$$\text{Maximize} \quad \sum_{i, \tau} q_{i\tau} \lambda_i(q_{i\tau}) - \sum_{\tau} c_{\tau} a_{\tau} \quad (4)$$

wherein:

$i = 1, \dots, I$ : customer class;

$\tau = 1, \dots, T$ : time periods, each of length  $\Delta$ ;

$\delta_{\tau}$  is tolerance on capacity violation in period  $\tau$ ;

$C_\tau$  is cost per unit of buying new capacity in period  $\tau$ ;

$d_i$  is duration of contract for customer class  $i$ ;

$D_i$  is actual duration of contract ( $d_i \Delta$ ) for customer class  $i$ ;

$n_{i\tau}$  is a number of existing contracts of type  $i$  still active at start of period  $\tau$ ;

$L_{i\tau}$  is a lower bound on contract price;

$U_{i\tau}$  is an upper bound on contract price;

$b_\tau$  is bandwidth available in period  $\tau$ ;

$a_\tau$  is bandwidth purchased by re-seller in period  $\tau$ ;

$q_{i\tau}$  is price to new or renewing customers for a new standard length contract of type  $i$  in period  $\tau$ ; and

$\lambda_i(q_{i\tau})$  is expected number of new customers of type  $i$  arriving in any period if a price for a contract is set at  $q_{i\tau}$ .

3. A method for optimizing pricing and capacity for bandwidth management using a computer as recited in claim 1 wherein said computer solving and optimization problem is running a non-linear programming software.

4. A computer readable medium comprising software for causing a computer to execute steps for optimizing pricing and capacity for bandwidth management, comprising the steps of:

receiving a mean and a variance of real usage for

each of a plurality of customer classes;  
 receiving price and demand curve data which  
 determines an arrival rate for each customer class;  
 receiving a number of existing customers in each  
 customer class;  
 receiving a bandwidth wholesale cost;  
 generating a computer model for an optimization  
 problem subject to a plurality of predetermined chance  
 constraints;  
 solving said optimization problem using said  
 computer to determine an amount of bandwidth to be  
 purchased in a time period at a given price for an  
 expected number of new customers in order to maximize  
 profit; and  
 outputting said amount of bandwidth to be  
 purchased and said expected number of new customers.

5. A computer readable medium comprising software for  
 causing a computer to execute steps for optimizing  
 pricing and capacity for bandwidth management as  
 recited in claim 4 wherein said plurality of  
 predetermined chance constraints comprises:

$$b_{\tau} = b_{\tau-1} + a_{\tau} \quad (\tau = 1, \dots, T) \quad (1)$$

$$L_{i\tau} \leq q_{i\tau} \leq U_{i\tau} \quad (i = 1, \dots, I; \tau = 1, \dots, T) \quad (2)$$

and said optimization problem comprises:

wherein:

$$\begin{aligned} & \sum_{i|\tau < d_i} [\lambda_{i\tau} \Delta \mu_i^2 + (n_{i\tau} + \lambda_{i\tau} \Delta)^2 \sigma_i^2 + (n_{i\tau} + \lambda_{i\tau} \Delta)^2 \mu_i^2] \\ & + \sum_{i|\tau \geq d_i} [\lambda_{i\tau} D_i (\mu_i^2 + \sigma_i^2)] + (\lambda_{i\tau} D_i \mu_i)^2] - \delta_\tau b_\tau^2 \leq 0 \quad \forall \tau \end{aligned} \quad (3)$$

$$\text{Maximize} \quad \sum_{i,\tau} q_{i\tau} \lambda_i(q_{i\tau}) - \sum_{\tau} C_\tau a_\tau \quad (4)$$

$i = 1, \dots, I$ : customer class;

$\tau = 1, \dots, T$ : time periods, each of length  $\Delta$ ;

$\delta_\tau$  is tolerance on capacity violation in period  $\tau$ ;

$C_\tau$  is cost per unit of buying new capacity in period  $\tau$ ;

$d_\tau$  is duration of contract for customer class  $i$ ;

$D_i$  is actual duration of contract ( $d_i \Delta$ ) for customer class  $i$ ;

$n_{i\tau}$  is number of existing contracts of type  $i$  still active at start of period  $\tau$ ;

$L_{i\tau}$  is a lower bound on contract price;  
 $U_{i\tau}$  is an upper bound on contract price;  
 $b_\tau$  is bandwidth available in period  $\tau$ ;  
 $a_\tau$  is bandwidth purchased by re-seller in period  
 $\tau$ ;

$q_{i\tau}$  is price to new or renewing customers for a  
new standard length contract of type  $i$  in period  $\tau$ ; and  
 $\lambda_i(q_{i\tau})$  is expected number of new customers of type  
 $i$  arriving in any period if a price for a contract is  
set at  $q_{i\tau}$ .

6. A computer readable medium comprising software for  
causing a computer to execute steps for optimizing  
pricing and capacity for bandwidth management as  
recited in claim 4 wherein said computer solving and  
optimization problem is running a non-linear  
programming software.